



Volume and surface area of 3D objects



TB27331

Content

- To introduce the topic, students will revisit the topics of finding volume and surface area of other three-dimensional objects, including cubes, spheres, and hemispheres. In that introduction, students explain the difference between finding the volume and surface area of a given object. Then use the dissectible sphere to give students a visual representation of a sphere and a hemisphere. Students will be able to see that a sphere has only one curved area, while a hemisphere has both a curved and a flat area. That discovery will help students understand that finding the volume of a hemisphere means to find half the volume of a sphere, and to find the surface area requires an additional step.
- This lesson assumes students have prior knowledge about how to find the volume and surface area of three-dimensional objects, including cubes and spheres.
- In this lesson plan volume equals V , surface area equals S , and area equals A .

Objectives

Students will...

- Explain the difference between volume and surface area
- Calculate the volume and surface area of hemispheres
- Compare the volume and surface areas of spheres and hemispheres

Materials

- Dissectible Sphere ([TB27331](#))
- Worksheet and answer key (included with lesson plan download)

Common core state standards

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

CCSS.Math.Content.7.G.B.4 — Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

CCSS.Math.Content.7.G.B.6 — Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

CCSS.Math.Content.8.G.C.9 — Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Introduction (5 mins.)

1. Ask students what it means when you ask for the volume of a three-dimensional object (*the amount of space **within** that given object*). Ask what it means when you ask for the surface area of a three-dimensional object (*the area of the surface on the **outside** of the given object*).
2. Have students look at a cube where the length, width, and height are all 9 cm. (**Figure 1**) Ask how to find the volume of the cube (*multiply length x width x height*). Students should multiply $9 \times 9 \times 9$ and get 729 cm^3 . Tell students that it is important to remember when working with volume, the measurement is in units cubed because they are finding the amount the three-dimensional shape can hold.
3. To find the surface area of this cube, students will first need to find the area of one of the faces. Ask how they should do this (*multiply length x width of one side*). Once they have done this, ask what answer they came up with (81 cm^2) and why it is in square units instead of units cubed (*one face of the cube is two dimensional while the entire cube is three dimensional*). To find the total surface area of the cube, students should be able to tell you that they need to multiply the area of one face by the total number of faces on the cube, which is 6 because there are 6 total faces on the cube. Have students do this. They should get a total surface area of 486 cm^2 .

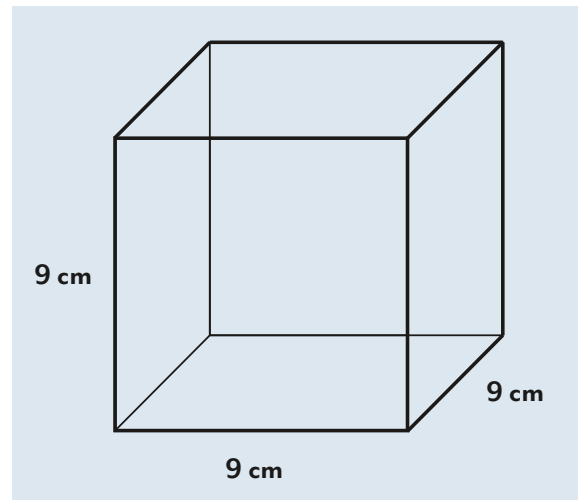


Figure 1

4. Show students the dissectible sphere fully intact and tell them that the radius of the sphere is 6 cm. Ask students what the formula is that determines the volume of a sphere ($V = \frac{4}{3} \pi r^3$). Have students use the formula to determine the volume of the sphere.
$$V = \frac{4}{3} \pi 6^3$$
$$V = \frac{4}{3} \pi 216$$
$$V = 288\pi \text{ or } \approx 904.78 \text{ cm}^3$$
Remind students that you've found how many cubic centimeters there are *inside* the sphere. Use the dissectible sphere to further illustrate the concept.
5. Now that they've determined the volume inside of the sphere, it's time to move to the outside. Ask what students notice about a sphere that makes it different than other three-dimensional shapes, such as a cube or a cone (*the entire surface is curved with no flat faces anywhere on the shape*). Have students keep that in mind as they continue to explore spheres and explore hemispheres later. Ask what the formula is for determining the surface area of a sphere ($S = 4\pi r^2$). Check with students to see if they remember why they use centimeters squared instead of centimeters cubed for surface area (*they are only looking at the surface of the sphere, which is two dimensional*). Use the dissectible sphere to further illustrate this concept to students. Have students use the surface area formula to determine the surface area of this sphere with a 6 cm radius.
$$S = 4\pi 6^2$$
$$S = 4\pi 36$$
$$S = 144\pi \text{ or } \approx 452.39 \text{ cm}^2$$

Check for understanding 1

Hand out worksheet, then ask students to solve the first two problems. Those problems ask students to find both the volume and the surface area of a cube and a sphere. These two three-dimensional shapes have been chosen because one asks students to find the area of flat surfaces (the cube) and one asks students to find the surface area of a curved surface (the sphere). During the lesson, students will need to find the surface area of a hemisphere, which is a combination of a flat surface and a curved surface. Once students have had a chance to solve the first two problems, ask the following questions:

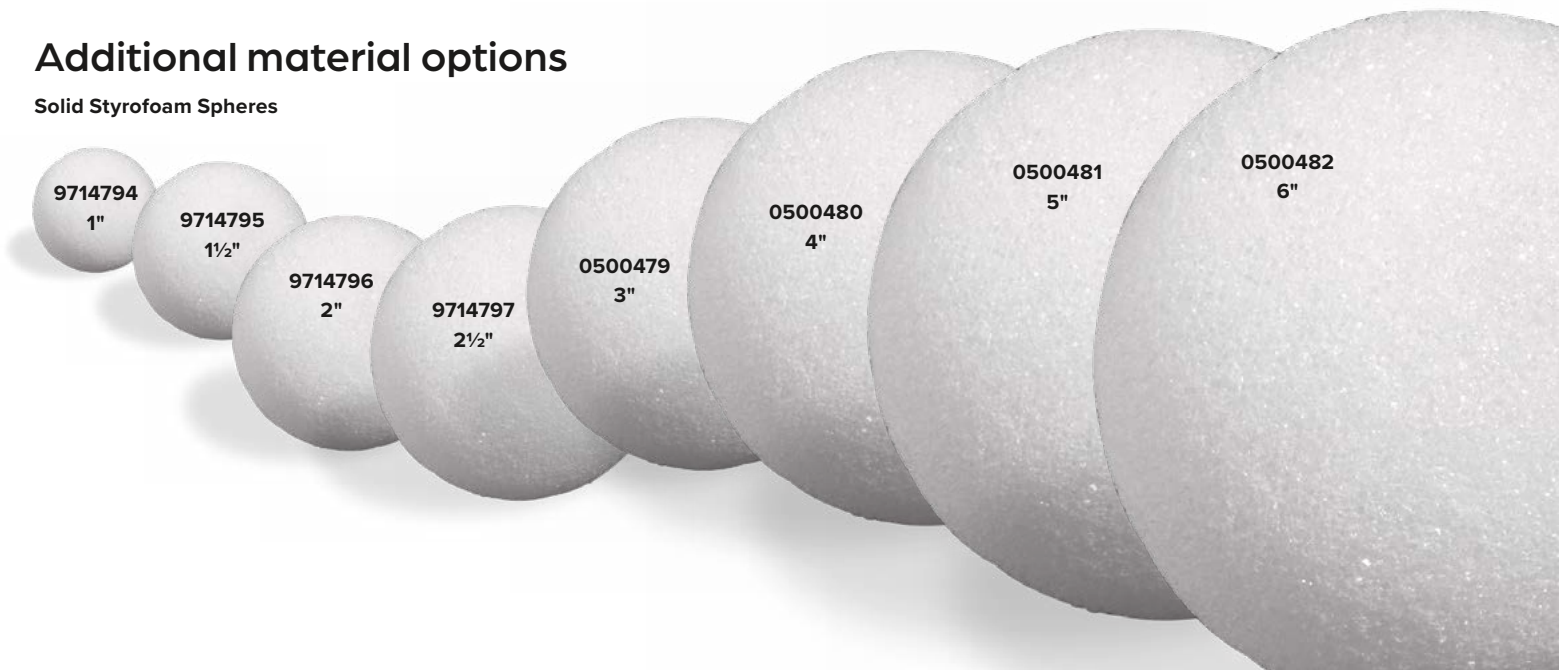
1. How do I find the volume of this sphere? ($V = \frac{4}{3} \pi r^3$)
2. What is r^3 ? ($7 \times 7 \times 7 = 343 \text{ in}^3$)
3. Simplify what remains to get $V = ?$ ($457\frac{1}{3} \pi$ or $\approx 1,436.76 \text{ in}^3$)
4. How do I find the surface area of this sphere? ($S = 4\pi r^2$)
5. What is r^2 ? ($7 \times 7 = 49 \text{ in}^2$)
6. Simplify what remains to get $A = ?$ (196π or $\approx 615.75 \text{ in}^2$)
7. What is the volume of the cube in problem 2? ($3.5 \times 3.5 \times 3.5 = 42.875 \text{ in}^3$)
8. What is the surface area of the cube? ($3.5 \times 3.5 = 12.25 \text{ in}^2$ for surface area of one face; $12.25 \times 6 = 73.5 \text{ in}^2$)

Activity

1. Students have now had a chance to find the volume and surface area of a three-dimensional figure with a curved surface. They have also had a chance to find the volume and surface area of a three-dimensional figure with flat surfaces. Tell students that they are now going to move on to another three-dimensional figure, the hemisphere. Take apart the dissectible sphere to show students the hemisphere. Turn it around and show it to them from multiple angles. Have students get together with a partner for a moment or two to talk about ways in which the hemisphere is both similar and different to a sphere, and how it is similar and different to a cube.
2. Get students back together as a group and have them tell you what they have come up with. Students should be able to conclude that a hemisphere is like a sphere because it has a curved surface. It differs from a sphere because it has a flat base and is half of a sphere. A hemisphere is like a cube because it has a flat base. It differs from a cube because it has a curved surface.
3. Have students remind you again of the formula they need to use to find the volume of an entire sphere ($V = \frac{4}{3} \pi r^3$). Ask what they think they need to do with this formula in order to make it applicable to a hemisphere (*half it because a hemisphere is half of a sphere*). Tell students since you are holding exactly half of a sphere, they can assume the volume or the amount of space within the hemisphere is also exactly half of a sphere. Ask what half of $\frac{4}{3}$ is ($\frac{2}{3}$). With that information, see if students can come up with the formula they will need to find the volume of a hemisphere ($V = \frac{2}{3} \pi r^3$).
4. Have students determine the volume of the hemisphere if the radius is 2.6 inches.
 $V = \frac{2}{3} \pi 2.6^3$
 $V = \frac{2}{3} \pi 17.576$
 $V = 35.152\pi$
 $V = 36.81 \text{ in}^3$
5. To find the volume of a hemisphere, students simply had to use half the formula they use to find the volume of a sphere. Wonder aloud if the same can be done to find the surface area of a hemisphere. Ask students what the surface area of a sphere is ($S = 4\pi r^2$) and what half of that formula would be ($S = 2\pi r^2$). Get students to discuss whether this new formula would work for finding the surface area of a hemisphere.
6. Take students back to the discussion you had at the beginning of the activity about the similarities and differences between a sphere and a hemisphere. Remind them that they realized that a hemisphere has a base and a sphere does not. Remind them that a hemisphere is a combination of a flat surface and a curved surface while a sphere is simply a curved surface. If they simply halve the formula for the surface area of a sphere, they will only be finding the surface area of the curved surface of the hemisphere. Show them this using the dissectible sphere, then ask what they are missing if they only do this (*the circular base*). Ask how to find the area of a circle ($A = \pi r^2$). Tell students that in order to find the complete surface area of a hemisphere, they need to add the surface area of the curved surface plus the surface area of the base. Take the two ($2\pi r^2 + \pi r^2$) and simplify the like terms to create the final formula ($S = 3\pi r^2$).
7. Use the final formula to find the surface area of the hemisphere, remembering that the radius is 2.6 inches.
 $S = 3\pi 2.6^2$
 $S = 3\pi 6.76$
 $S = 20.28\pi \text{ or } 63.71 \text{ in}^2$
8. Use the yellow insert to dissect the hemisphere in half, then tell students that this time, they will be finding the surface area of this quarter sphere. Ask how they would go about doing that. Facilitate a discussion around this question. Students should be able to conclude that you simply need to divide the surface area of the hemisphere in half. When they calculated the surface area of the hemisphere, they accounted for a curved surface and a flat surface. The quarter sphere also has a curved surface and a flat surface, so they could simply divide by 2.

Additional material options

Solid Styrofoam Spheres



Check for understanding 2

Create a hemisphere using the dissectible sphere model, then have students look back at the sphere in problem 1 of the worksheet. Ask the following questions:

1. If we split the sphere in problem 1 into two hemispheres, what will the radius of each hemisphere be? (Each radius will still be 7 inches.)
2. Pretend the radius of the hemisphere in my hand is 7 inches. What did we determine the volume of the entire sphere is? ($\approx 1,436.76 \text{ in}^3$)
3. What do we need to do to determine the volume of one hemisphere of this sphere? (Divide the sphere's volume by 2. The volume of the hemisphere is $\approx 718.38 \text{ in}^3$.)
4. Now let's find the surface area. Remember we use $S = 4\pi r^2$ to find the surface area of the entire sphere. What do we accomplish by halving that formula? (We find the surface area of the curved face of the hemisphere.)
5. What is missing? (The surface area of the flat face. We find the area with the formula $A = \pi r^2$.)
6. What is the entire formula to find the surface area of a hemisphere? ($2\pi r^2 + \pi r^2$, which simplifies to $S = 3\pi r^2$.)
7. Use the formula to determine the surface area of this hemisphere with a radius of 7 inches. ($A \approx 461.81 \text{ in}^2$)
8. Find the volume of the quarter spheres. ($V \approx 359.19 \text{ in}^3$)
9. Find the surface area of the quarter sphere. ($A \approx 230.91 \text{ in}^2$)

Have students complete the remainder of the worksheet independently.

Intervention

Provide each student (or small group of students) with a dissectible sphere. Allow them to investigate the sphere in front of them while you show the dissectible sphere to the whole class. They can dissect their own spheres to better understand and visualize the concepts discussed as you work through the lesson.

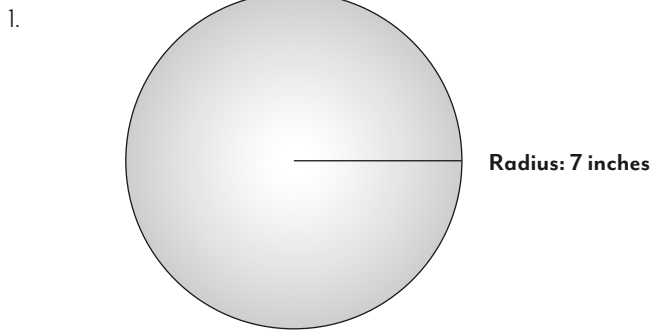
Extension

Change the information students are provided with at the start of a problem. Give students the volume of a particular sphere or hemisphere. Ask them to determine the surface area and radius of that figure.

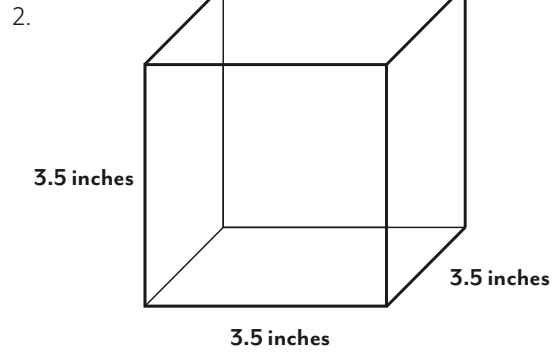


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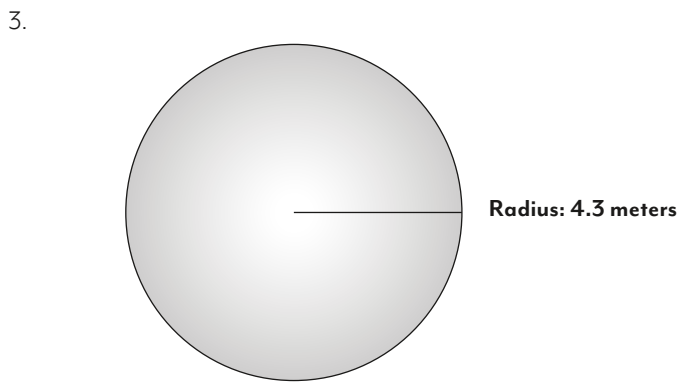
Directions: Find the volume and surface area of each shape.



Volume = _____
Surface Area = _____

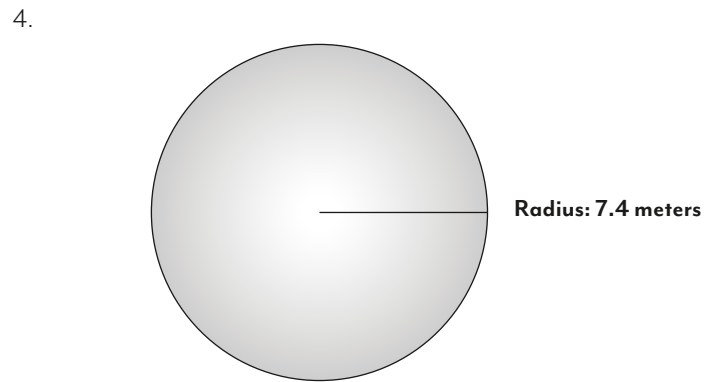


Volume = _____
Surface Area = _____



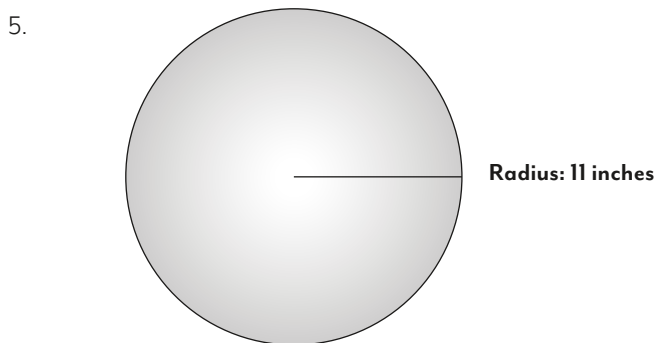
Volume = _____
Surface Area = _____

Split the sphere into two hemispheres.
What is the volume of one hemisphere? _____
What is the surface area of one hemisphere? _____



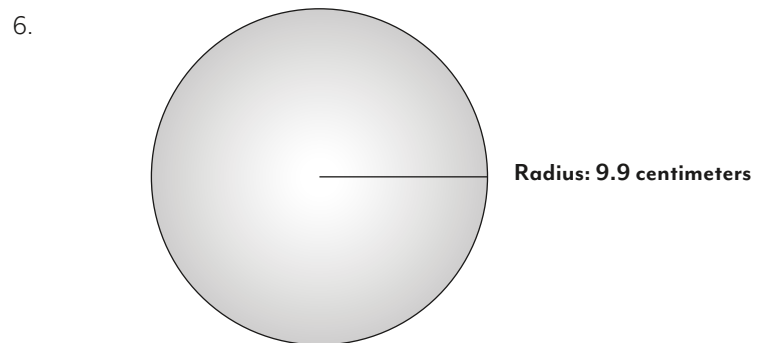
Volume = _____
Surface Area = _____

Split the sphere into two hemispheres.
What is the volume of one hemisphere? _____
What is the surface area of one hemisphere? _____



Volume = _____
Surface Area = _____

Split the sphere into two hemispheres.
What is the volume of one hemisphere? _____
What is the surface area of one hemisphere? _____

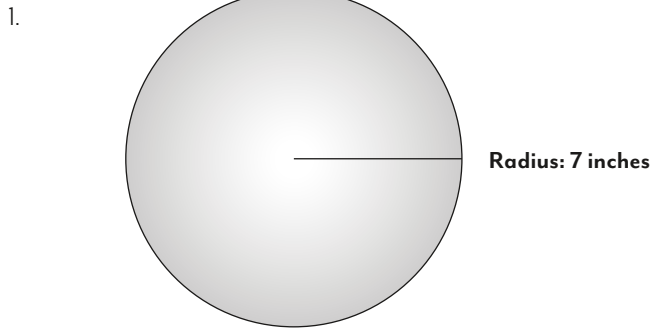


Volume = _____
Surface Area = _____

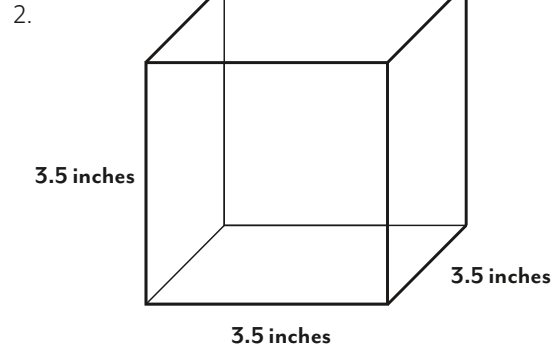
Split the sphere into two hemispheres.
What is the volume of one hemisphere? _____
What is the surface area of one hemisphere? _____

Name: _____ Date: _____

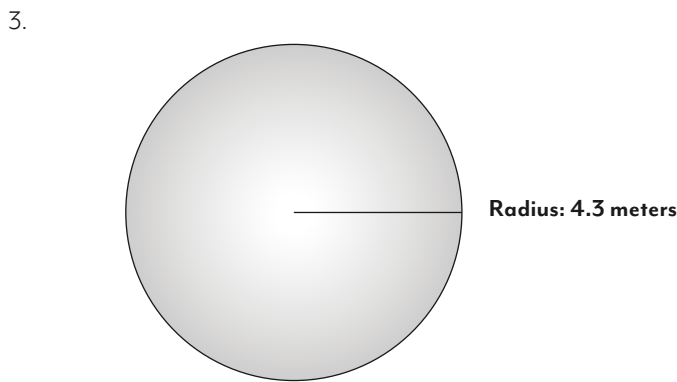
Directions: Find the volume and surface area of each shape.



Volume = $457\frac{2}{3}\pi$ or $\approx 1,436.76 \text{ in}^3$
 Surface Area = 196π or $\approx 615.75 \text{ in}^2$

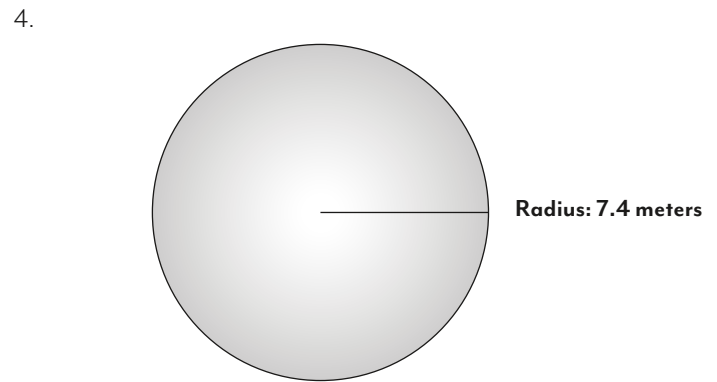


Volume = 42.875 in^3
 Surface Area = 73.5 in^2



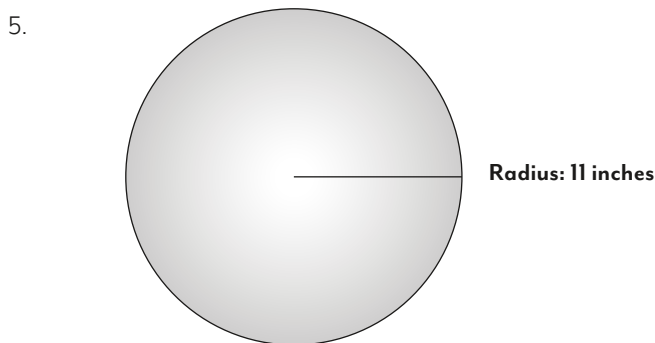
Volume = 106π or $\approx 333.04 \text{ m}^3$
 Surface Area = 73.96π or $\approx 232.35 \text{ m}^2$

Split the sphere into two hemispheres.
 What is the volume of one hemisphere? 53π or $\approx 166.5 \text{ m}^3$
 What is the surface area of one hemisphere? 55.47π or $\approx 174.26 \text{ m}^2$



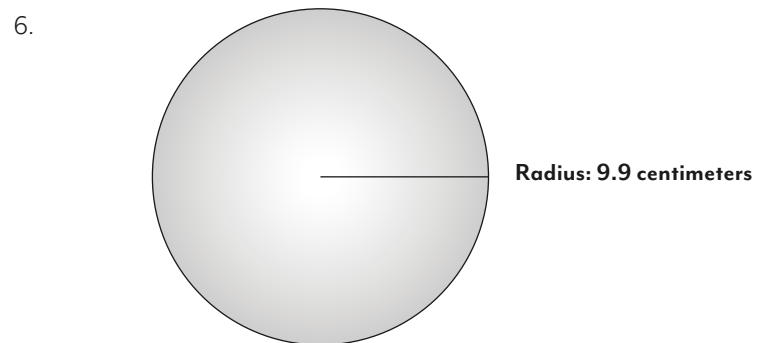
Volume = 540.3π or $\approx 1,697.4 \text{ m}^3$
 Surface Area = 219.04π or $\approx 688.13 \text{ m}^2$

Split the sphere into two hemispheres.
 What is the volume of one hemisphere? 270.15π or $\approx 848.70 \text{ m}^3$
 What is the surface area of one hemisphere? 164.28π or $\approx 516.1 \text{ m}^2$



Volume = $1,774\frac{2}{3}\pi$ or $\approx 5,575.25 \text{ in}^3$
 Surface Area = 484π or $\approx 1,520.53 \text{ in}^2$

Split the sphere into two hemispheres.
 What is the volume of one hemisphere? $887\frac{1}{3}\pi$ or $\approx 2,787.64 \text{ in}^3$
 What is the surface area of one hemisphere? 363π or $\approx 1,140.4 \text{ in}^2$



Volume = $1,293.73\pi$ or $\approx 4,064.38 \text{ cm}^3$
 Surface Area = 392.04π or $\approx 1,231.63 \text{ cm}^2$

Split the sphere into two hemispheres.
 What is the volume of one hemisphere? 646.87π or $\approx 2,032.19 \text{ cm}^3$
 What is the surface area of one hemisphere? 294.03π or $\approx 923.72 \text{ cm}^2$