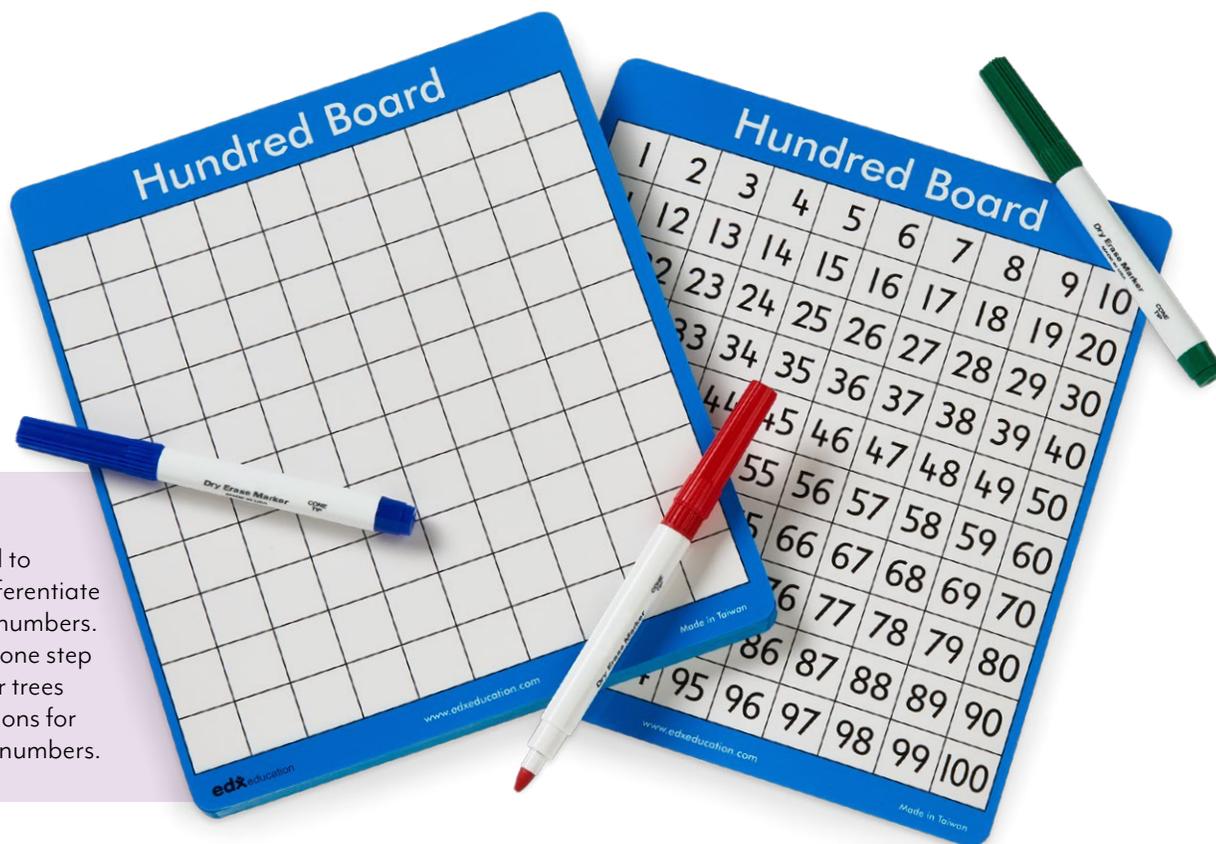Developed with Kristin Hotter

Prime and composite numbers

Volume 23 | Gr. 4-6

Time: One 60-minute lesson or two 30-minute lessons

Content

Use a hundreds board to identify, mark, and differentiate prime and composite numbers. Using that tool, move one step further to create factor trees and build the foundations for prime factorization of numbers.

Objectives

Students will...
- Create factor trees, allowing them to identify the prime factorization of a given number up to 100
- Differentiate between prime and composite numbers
- Deconstruct composite numbers into their prime factors

Materials
- Hundred boards - Pack of 30 (**TB26166**)
- 3 different colored dry-erase markers (**TB21489** Green, **TB21490** Blue, and **TB21492** Red - Box of 30)
- Worksheet and answer key (attached with lesson plan download)

Common core state standards

CCSS.Math.Content.4.OA.B.4 — Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

CCSS.Math.Content.6.NS.B.4 — Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9+2).*

Introduction

1. Make sure that each student has a hundreds board and three different-colored markers. Begin by telling students about how they have spent time learning their multiplication tables, that they can quickly tell you what 6 x 2 or 8 x 8 is, and that they will now take what they know about multiplication and go a few steps further by using those multiplication facts to help them differentiate between numbers that are prime and numbers that are composite.

2. Before continuing, it is necessary for students to understand what prime and composite numbers are. Explain that composite numbers are numbers that have factors other than one and themselves. For an example, ask what two numbers can be multiplied together to get 4 *(1 x 4 and 2 x 2)*. Say that if 1 x 4 was the only pair of numbers that could be multiplied together to get 4, it would be a prime number, but since 2 x 2 can also be multiplied together to make 4, 4 is a composite number, since 4 has the factors of 1, 2, and 4.

3. Explain that prime numbers are numbers that only have 1 and themselves as factors. Use the number 7 as an example. Ask what numbers can be multiplied together to get 7. Only 1 x 7 makes 7, so since 1 and 7 are 7's only factors, 7 is a prime number.

4. Now that students have a definition for each type of number, they can put that into action along with what they already know about multiplication to give them a better understanding of the numbers. Tell students that they will be using their markers to fill in different squares on their hundreds board. They need to select one of their colors to indicate composite numbers on the board. They will use that marker to systematically color in all the composite numbers on the board.

5. Tell students that they will begin with multiples of 2. Ask what the first multiple of 2 is *(2)*, then ask what two numbers can be multiplied to equal 2 *(1 and 2)*. Since that is 1 and the number itself, say that they can't be certain if 2 is prime or composite at this point, so they will skip that number for now.

6. Ask what the next multiple of 2 is *(4)*. Say that they can color in 4 because 2 is a factor other than 1 and the number itself *(in this case, 4)*. Move on to 6, then 8, and so on, using the same line of questioning, until students see that every even number from 4-100 should be colored as composite. Once they have that understanding, let them color the remaining squares. As they do so, be sure to circulate to ensure that everything has been colored correctly.

7. Say that now that all of the multiples of 2 are colored, they will now look at multiples of 3. Ask what the first multiple of 3 is *(3)*, then ask what can be multiplied to equal 3 *(1 and 3)*. Say that they will not color in 3 at this point, as they still need to determine if it is prime or composite, then ask what multiple of 3 comes next *(6)*. Ask what they notice about 6 *(it's already colored)*. Why is it already colored? *(It is a multiple of both 2 and 3)*.

8. Point out that, as they continue on, they will see that some of the multiples will already be colored. This is OK, as it simply means that those numbers are multiples of several factors.

9. Ask what multiple of 3 comes next *(9)*. Continue on with this line of questioning until students see that every third number from 9-100 will be colored as composite. Once they have that understanding, let them color the remaining squares as you circulate to ensure that everything has been colored correctly.

10. Now that all the multiples of 2 and 3 have been colored, have them move on to multiples of 4. They should notice that 4 is already colored. When asked about the next multiple of 4, they should know it is 8 and see it is also already colored. Have them go through the multiples of 4 and see if there are any that are not colored yet. They will observe that all the multiples of 4 are already colored. When asked why, they should be able to tell you that since 4 is also a multiple of 2, all numbers that are multiples of 4 are also multiples of 2.

11. Move on to multiples of 5. Ask what the multiples of 5 are *(1 and 5)*. Since this is the number and itself, students should skip 5 for now until they are sure if it is prime or composite. Ask what they know to be true about all multiples of 5 *(they either end in a 5 or a 0)*, then ask what they notice about all the numbers on their hundreds board that have a 0 in the ones place *(they're already colored)*. Are there any numbers with 5 in the ones place that still need to be colored? *(Yes)*. Have them color all those numbers except for 5 itself.

12. Move on to multiples of 6. Students should notice that 6 is already colored. Have them look at the first few multiples of 6. They will see that all of those numbers are also colored. Ask why it seems that all the multiples of 6 are already colored *(they are all multiples of 2 and 3 as well)*. Give students a minute to double check all the multiples of 6 to make sure that they are all colored. All the multiples should already be colored, but this is a good way to double check.

13. Move on to multiples of 7. Students should be able to say that they can multiply 1 and 7 together to get 7. As with 3 and 5, they should skip 7 until they can be sure if 7 is prime or composite. Look at the next two multiples of 7: 14 and 21. Students will notice both of these multiples are already colored in. However, tell students that since 7 itself wasn't colored in, there may be other multiples of 7 that aren't colored in, so they should go through their hundreds board and color in any multiples of 7 that aren't currently colored in. Give them a minute or two to do so, then ask if there were any other numbers that needed to be colored in *(49 and 91)*.

14. Point out the last three numbers in the top row of the hundreds board: 8, 9, and 10. Ask students what they notice about those three numbers *(they are already colored)*. Ask what that tells them about all the multiples of 8, 9, and 10 *(they are already colored, too)*.

15. Have students set aside their composite color marker, then point out that there are still quite a few numbers on the board that are not colored in. Direct their attention to the number 2. Ask what the factors of 2 are *(1 and 2)*. Since there aren't any others, they know that 2 is a prime number. Do the same with 3 *(factors are 1 and 3)*. Since there are no other factors for 3, they know that 3 is also a prime number. Have them select one of the other colors of markers and color in 2 and 3. This color will be for coloring in prime numbers.

16. Continue with this line of questioning for the rest of the prime numbers. Those numbers will be: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

17. There should be one number left uncolored on the hundreds board: 1. Ask what numbers are factors of 1 *(1 is the only factor)*. Tell students that since 1 only has one factor, it is the only positive number that is neither prime nor composite. Therefore, it will get a color all of its own. Have students use their third color to shade in 1 on their hundreds board. Students should keep the hundreds board for reference as they look more closely at different numbers and try to break them down into their prime factors. Do the Quick Check on the next page with students. If you wish to teach the lesson over two class periods, this would be a good place to stop.

Quick check

1. Is 67 prime or composite? *(prime)*
2. Is 99 prime or composite? *(composite)*
3. Is 56 prime or composite? *(composite)*
4. Is 23 prime or composite? *(prime)*

Activity 1

1. Students have taken some time to determine which numbers between 2 and 100 are prime and which are composite. Now they will take that one step further to break those composite numbers down into their prime factors. Tell students that this process is called prime factorization, and it is useful when they spend time finding the greatest common factor and least common multiple of different groups of numbers. For now, they will just focus on the prime factorization part.

2. Tell students that when they determine the prime factorization of a number, they break it down into its multiples until they only have prime numbers. You will go through several examples together, beginning with the number 18. In order to determine its prime factorization, you will make a factor tree. Explain that a factor tree has the original number, in this case 18, at the top, and branches that come from that number until the prime factors are reached.

3. Say that there are a few different pairs of factors for the number 18, those being 1 and 18, 2 and 9, and 3 and 6. When determining a number's prime factorization, they should never use the pair containing 1 and the number itself, as that pair will not help them break down the number at all. Say that you will choose to use the 3 and 6 pair of factors instead. Create two branches that come from the bottom of the 18, then write 3 at the end of one branch and 6 at the end of the other.

4. Students should now refer to their prime/composite hundreds board to determine if either of these factors is prime. They should see that 3 is prime and 6 is not. Circle the 3 and tell students that you are doing so because it is prime. The circle indicates that they are done with that set of branches. Since 6 is not prime, draw two more branches from it, as it can be broken down further.

5. State the factors for 6: 1 and 6 and 2 and 3. Ask which pair of factors you should use *(2 and 3 because you never choose 1 and the number)*. Write 2 and 3 at the ends of the new branches on the factor tree, then ask if either of these numbers are prime. Students may refer to their hundreds board. They should determine both 2 and 3 are prime numbers. Circle both numbers. Remind students that circling the number indicates that they can go no further in breaking down the number and creating branches. Since all numbers have now been circled, they have completed their prime factorization of 18 **(see figure 1)**.

6. Ask what numbers have been circled *(2, 3, 3)*. Say that the prime factorization of 18 is 2 x 3 x 3, or 2×3^2. Guide students through checking this to see if it is true *(2 x 3 = 6, and 6 x 3 = 18)*. Since the final answer is the original number, they know their prime factorization is correct.

7. Write the number 24 on the board, then ask for two numbers that are not 1 or 24 that can be multiplied together to get 24 *(6 and 4, 8 and 3, or 2 and 12)*. Select 8 and 3, and put those as the two original branches coming down from 24. Ask if either of these numbers are a prime number *(3 is a prime number and 8 is not)*. Ask what should be done next with these two numbers *(3 should be circled, and 8 should have two branches drawn coming down from it)*.

8. Since 8 is not a prime number, students should know that there are factors other than 1 and 8 that can be multiplied together to get 8. They should supply the factors 2 and 4. Write those on the factor tree, then ask if either of these numbers is a prime number *(2 is and 4 is not)*. Students should tell you to circle the 2 and draw two branches coming down from the 4.

9. Continue the same process with the 4 (identifying factors that can be multiplied that are not 1 and 4, writing 2 and 2 on the tree, identifying 2 as a prime number, and circling the 2s). Remind students that once all the factors are circled as they are now, that means the prime factor tree has been completed and it is time to determine the prime factorization for 24 **(see figure 2)**.

10. Have students list what numbers on the prime factor tree are circled *(3, 2, 2, 2)*. Say that they should start prime factorization with the smallest factor, so the prime factorization for 24 is 2 x 2 x 2 x 3, or $2^3 \times 3$. Guide students through double checking the answer by multiplying all the prime factors together *(2 x 2 = 4, 4 x 2 = 8, 8 x 3 = 24)*. Since the final answer is the original number, that indicates that they did their prime factorization correctly.

11. Work one more problem together. Write the number 12 on the board, then have students provide factors of 12 that are not 1 or 12 *(2 and 6 or 3 and 4)*. Point out that for the previous prime factor trees you've done together, you have only used one pair of factors. This time, they will conduct an experiment and see what happens when different factors are chosen. Have half the class break down 12 using 2 and 6, and the other half of the class break down 12 using 3 and 4. Give students a few minutes to write down their factorizations, then select a student from each half of the class to come up to the board and draw their completed tree **(see figure 3 on the next page)**. Have students compare the factorizations so they can realize that the results are the same even though a different pair of factors was used initially. Explain to students that it doesn't matter what pair of factors they start with for a number because the end result will always be the same. There is more than one way to solve the same problem. If students need more practice, have them find the prime factorization of 24 using a different pair of factors than 8 and 3.
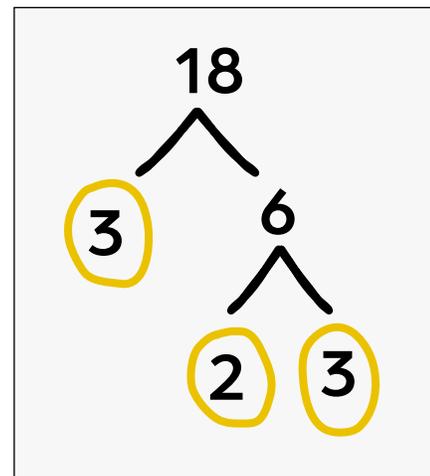
Factor trees for 18

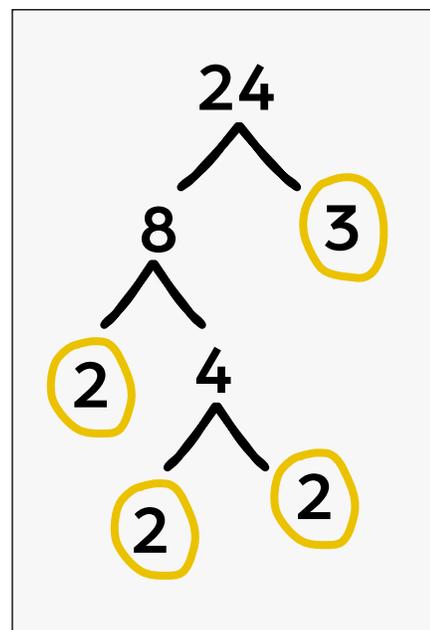

Figure 1

Factor trees for 24

Figure 2

Activity (continued)

12. Distribute the worksheet, then explain that it contains more composite numbers to break down into prime factors. Have them look at the first number, 28. Ask the class what are two factors of 28 that are not 1 and 28 *(4 and 7 or 2 and 14)*. Give students a few minutes to draw a prime factorization tree for 28, then ask the questions in the Check for Understanding below.

13. Point out once again that, no matter which set of factors they begin with, the end result will be the same. Guide students through double checking to make sure the factorization is correct *(2 x 2 = 4, 4 x 7 = 28)*. Since they came up with the original number for the final answer, the prime factorization is correct.

14. Explain to students that they will complete the rest of the worksheet independently. They should create a factor tree for each number, write the prime factorization out just like they have seen in the classroom examples, and check their work.

Factor trees for 12
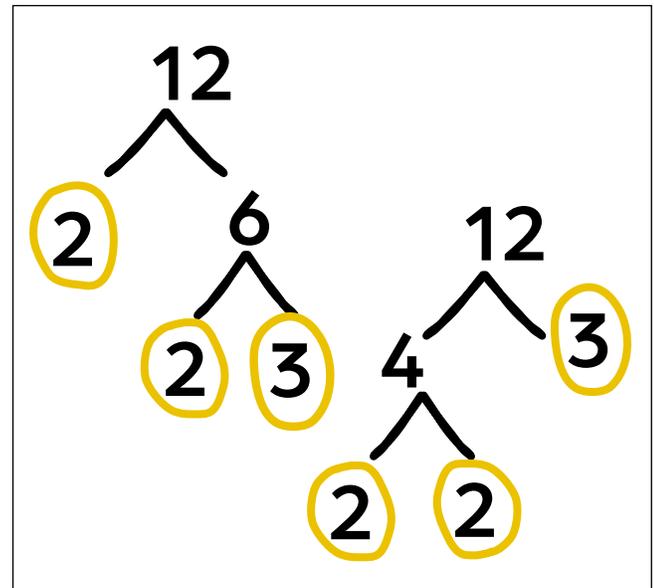
Figure 3

Check for understanding

Start with a student who used 4 and 7 as their factors

1. Is 4 a prime number? *(no)*
2. What did you have to do with 4? *(draw two branches coming down from it)*
3. Is 7 a prime number? *(yes)*
4. What did you have to do with 7? *(circle it)*
5. What are two factors of 4 *(2 times 2)*
6. Is 2 a prime number? *(yes)*
7. What did you do with 2? *(circle it)*
8. Do you have any numbers left to break down? *(no)*
9. What is the prime factorization for 28? *(2 x 2 x 7 or 2^2 x 7)*

Now ask a student who used 2 and 14 as their factors

1. Is 2 a prime number? *(yes)*
2. What did you have to do with 2? *(circle it)*
3. Is 14 a prime number? *(no)*
4. What did you have to do with 14? *(draw two branches coming down from it)*
5. What are two factors of 14 *(2 and 7)*
6. Is 2 a prime number? *(yes)*
7. What did you do with 2? *(circle it)*
8. Is 7 a prime number? *(yes)*
9. What did you do with 7? *(circle it)*
10. Do you have any numbers left to break down? *(no)*
11. What is the prime factorization for 28? *(2 x 2 x 7 or 2^2 x 7)*

Intervention

Use only even numbers to begin practice. Have students always choose 2 as one of the factors of that even number. That will give them one branch that always ends with the first set of factors. They will only need to focus on one set of branches coming down from the number.

Extension

Have students explore with numbers greater than 100. Explain that they can use the factor rules they already know to determine starting factors for the given numbers. EXAMPLE: If the number is even, you know it's divisible by 2. If the number's digits add up to a multiple of 3, you know it's divisible by 3.
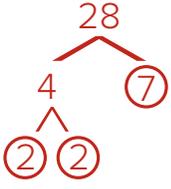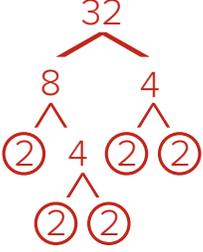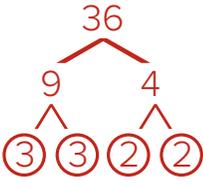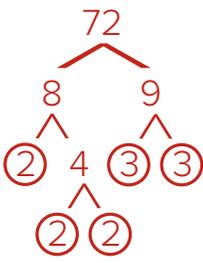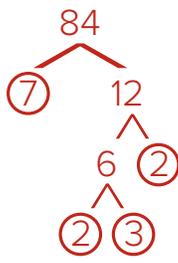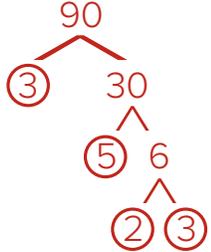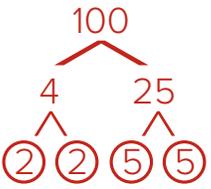
Name: _____   Date: _____

Directions: Create a factor tree for each number. Write the prime factorization for each number, then check your work.

Factor tree for 28	Prime factorization Check your work	Factor tree for 32	Prime factorization Check your work
Factor tree for 36	Prime factorization Check your work	Factor tree for 45	Prime factorization Check your work
Factor tree for 50	Prime factorization Check your work	Factor tree for 56	Prime factorization Check your work
Factor tree for 72	Prime factorization Check your work	Factor tree for 84	Prime factorization Check your work
Factor tree for 90	Prime factorization Check your work	Factor tree for 100	Prime factorization Check your work

NOTE: Factor trees may vary.

Factor tree for 28	Prime factorization
28 = 4 → (2)(2), 7	2 x 2 x 7 Check your work 2 x 2 = 4 4 x 7 = 28

Factor tree for 32	Prime factorization
32 = 8, 4 → 8 → (2), 4 → (2)(2); 4 → (2)(2)	2 x 2 x 2 x 2 x 2 Check your work 2 x 2 = 4 4 x 2 = 8 8 x 2 = 16 16 x 2 = 32

Factor tree for 36	Prime factorization
36 = 9, 4 → 9 → (3)(3); 4 → (2)(2)	2 x 2 x 3 x 3 Check your work 2 x 2 = 4 4 x 3 = 12 12 x 3 = 36
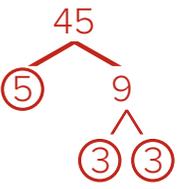
Factor tree for 45	Prime factorization
45 = (5), 9 → (3)(3)	3 x 3 x 5 Check your work 3 x 3 = 9 9 x 5 = 45
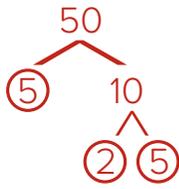
Factor tree for 50	Prime factorization
50 = (5), 10 → (2)(5)	2 x 5 x 5 Check your work 2 x 5 = 10 10 x 5 = 50
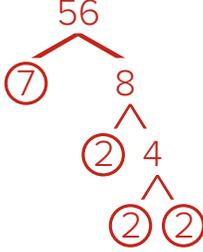
Factor tree for 56	Prime factorization
56 = (7), 8 → (2), 4 → (2)(2)	2 x 2 x 2 x 7 Check your work 2 x 2 = 4 4 x 2 = 8 8 x 7 = 56

Factor tree for 72	Prime factorization
72 = 8, 9 → 8 → (2), 4 → (2)(2); 9 → (3)(3)	2 x 2 x 2 x 3 x 3 Check your work 2 x 2 = 4 4 x 2 = 8 8 x 3 = 24 24 x 3 = 72

Factor tree for 84	Prime factorization
84 = (7), 12 → 6 → (2)(3); (2)	2 x 2 x 3 x 7 Check your work 2 x 2 = 4 4 x 3 = 12 12 x 7 = 84

Factor tree for 90	Prime factorization
90 = (3), 30 → (5), 6 → (2)(3)	2 x 3 x 3 x 5 Check your work 2 x 3 = 6 6 x 3 = 18 18 x 5 = 90

Factor tree for 100	Prime factorization
100 = 4, 25 → 4 → (2)(2); 25 → (5)(5)	2 x 2 x 5 x 5 Check your work 2 x 2 = 4 4 x 5 = 20 20 x 5 = 100